The Visual Computer
https://doi.org/10.1007/s00371-021-02281-5

ORIGINAL ARTICLE

Low dynamic range histogram equalization (LDR-HE) via quantized Haar wavelet transform

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Accepted: 5 August 2021

Abstract
Conventional contrast enhancement methods stretch histogram bins to provide a uniform distribution. However, they also stretch the existing natural noises which cause abnormal distributions and annoying artifacts. Histogram equalization should mostly be performed in low dynamic range (LDR) in which noises are generally distributed in high dynamic range (HDR). In this study, a novel image contrast enhancement method, called low dynamic range histogram equalization (LDR-HE), is proposed based on the Quantized Discrete Haar Wavelet Transform (HWT). In the frequency domain, LDR-HE performs a de-boosting operation on the high-pass channel by stretching the high frequencies of the probability mass function to the nearby zero. For this purpose, greater amplitudes than the absolute mean frequency in the high pass band are divided by a hyper alpha parameter. This damping parameter, which regulates the global contrast on the processed image, is the coefficient of variations of high frequencies, i.e., standard deviation divided by mean. This fundamental procedure of LDR-HE definitely provides a scalable and controlled dynamic range reduction in the histograms when the inverse operation is done in the reconstruction phase in order to regulate the excessive contrast enhancement rate. In the experimental studies, LDR-HE is compared with the 14 most popular local, global, adaptive, and brightness preserving histogram equalization methods. Experimental studies qualitatively and quantitatively show promising and encouraging results in terms of different quality measurement metrics such as mean squared error (MSE), peak signal-to-noise ratio (PSNR), Contrast Improvement Index (CII), Universal Image Quality Index (UIQ), Quality-aware Relative Contrast Measure (QRCM), and Absolute Mean Brightness Error (AMBE). These results are not only assessed through qualitative visual observations but are also benchmarked with the state-of-the-art quantitative performance metrics.

Keywords Image enhancement · Scalable contrast improvement · Quantized Discrete Haar Wavelet Transform · Quantized probability mass function

1 Introduction

Image enhancement and contrast improvement methods are extensively used in many image processing applications since the increased image quality is very crucial for human visual perception. Contrast can be described as the difference in visual properties that enables to distinguish an object from other objects and backgrounds. Namely, it is the difference in color intensity and luminance reflected from two adjacent surfaces. Since the human visual system is clearly more contrast sensitive than luminance, a human can easily perceive the environment by contrast.

As a restoration scheme, histogram equalization (HE) methods are used for contrast enhancement and visual quality improvement in many areas [1]. The main purpose of these methods is to optimize image contrast by distributing the frequencies of image intensity information uniformly. HE techniques basically are run in both spatial and frequency domains. Frequency zone is more preferable since it gives more accurate results.

Random variation of color intensities is called noise which can naturally appear in digital images. Noises caused by external sources are the degradation of color signals. Conventional HE techniques might normally create some artifacts and abnormal patterns in the processed image because of these existing noise elements. Noises in the input

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Published online: 19 August 2021

image are also equalized throughout the enhancement process [2]. Remaining artifacts and abnormal patterns reduce the image quality and natural look. Similarly, excessive contrast enhancement causes unnatural appearances. An ideal method should keep the originality and brightness mostly unchanged.

Most image contrast enhancement techniques fail in their duty by producing visual artifacts, giving more brightness, generating over-contrasted parts, creating abnormal patterns in the processed images. It is true that over enhancement in both brightness and contrast significantly cause annoying artifacts and unnatural appearances. An ideal method should enhance the contrast of the processed image while keeping the quality and brightness with respect to the original one. Hence, there is a fact that there should be a balanced histogram equalization.

Most of the contrast enhancement methods, even performed with the Haar Wavelet Transformation (HWT) models, can also preserve existing noises in the image [3]. HE should be performed in low dynamic range (LDR) since noises are naturally distributed in high dynamic range (HDR) [4, 5]. Namely, noises in an image can be generally found in the high frequencies.

In this study, a discrete HWT is empowered in LDR in order to improve the image quality by eliminating noises. As a novel contribution, LDR is satisfied with Quantized Discrete HWT [6] on histograms before equalization. Thus, LDR-HE (low dynamic range histogram equalization) by a quantized parameter is proposed to improve image quality using the Quantized Discrete HWT. In this method, existing noises are eliminated with an alpha parameter. This damping parameter used in the probability mass function (PMF) softens the high oscillations. This mechanism additionally provides a scalable contrast enhancement approach. This study also displays that the scalable quantization factor provides a great contribution in quality enhancement.

Like discrete cosine transform (DCT) and discrete Fourier transform, HWT is a lossless convolution technique which keeps the mean of the input vector as constant after de-convolution [7]. In this study, Quantized Discrete HWT is employed in order to divide the 1D histogram signals into two frequency components: low-frequency and high-frequency sub-bands. Since optimum HE requires contrast stretching, mean preserving property of HWT is used in this regard. Additionally, noises which mainly occur in the high-frequency band are reduced so that the quality decrement can be minimized. In order to reduce dynamic range in frequency domain, high-frequency information is quantized with a functional parameter of quantization factor which is similar to the DCT-quantization technique in JPEG compression. The concept of quantization factor derives from the DCT approach used in the JPEG compression. After the de-convolution procedure, a low dynamic range histogram equalization might be achieved with enhanced image quality and contrast.

This article has five more sections. The second chapter below covers a literature review. In the third chapter, there are detailed explanations of the presented approach. An extensive comparative study and discussions are placed into the fourth section. The summarized contributions are presented in the fifth section before the availability and acknowledgements section.

2 Literature review

Various HE (histogram equalization) algorithms for image quality achievement and contrast enhancement have been proposed previously in the literature. Some brightness preserving HE methods are also introduced. All these methods traditionally focus on contrast improvement by changing the color intensities. Some features and information in the image cannot be well preserved by these HE methods.

Numerous recent studies in the literature are evaluated carefully and only a few of them are found relevant and valuable to be examined and included into this article. These studies are categorized and presented as possible as in a chronological order.

The HE algorithms can be roughly examined in two main categories: global HE (GHE) and local HE (LHE). In GHE, the main purpose is to get a uniformly distributed histogram by performing a function for cumulative densities of the input image. This approach can be suitable for global enhancement but cannot preserve the local information and characteristics. LHE overwhelms these problems by using the color intensities of neighboring pixels with a kernel matrix. In other words, a mask sequentially visits the all pixels to find out the cumulative distribution. However, performing LHE needs higher computational cost. Besides, LHE occasionally generates excessive noises, over-enhancements, and checkerboard effects in some parts on the enhanced image [8].

The most well-known HE method is adaptive histogram equalization (AHE) which has a tendency to produce noises and over-amplifications in relatively homogeneous regions of the image. CLAHE (Contrast Limited AHE) is the modification of AHE which prevents these problems by limiting the amplification [9]. There are some improved versions of CLAHE in the literature. Some of them are the nonlinear and adaptive HE method using wavelet-based gradient histograms [10] and gradient mapping [11]. Another one is the Discrete Wavelet Transformation which has three phases [12]. Decomposing the original image into low and high-frequency components is the first phase. Enhancing the low-frequency coefficients and keeping high-frequency coefficients steady in order to suppress the noise are the second one. The proposed weighting factor which reduces over-enhancement is the last phase.

The exact histogram specification (EHS) method moves from the problem into a k-dimensional space by using an invertible cumulative distribution function (CDF) in order to induce a strict ordering among pixels. EHS generally focuses on ill-posed problems by using a probability density function and a predefined square mask [13].

A dynamic HE (DHE) is proposed by Al-Wadud et al. for low-contrast images by adjusting only one parameter. DHE subdivides the histogram with respect to local minimum and allocates gray-level ranges for each subdivision until equalizing them separately [14]. Another dynamic approach is the Brightness Preserving Dynamic Fuzzy HE (BPDFHE) which uses fuzzy image statistics [15]. This method performs the five-step stages: computing fuzzy histogram, partitioning of histogram, dynamically equalizing histogram of partitions, and normalizing brightness.

Mean preserving Bi-HE (BBHE) [16], Dualistic Sub-Image HE (DSIHE) [17], and Minimum Mean Brightness Error Bi-HE (MMBEBHE) [18] are also proposed to both improve contrast and maintain brightness. BBHE splits the histogram into two sub-histograms using a mean intensity threshold. Then, each section is equalized individually. In contrast, the histogram at the median is separated by DSIHE. MMBEBHE is based on the minimum absolute mean brightness error between output and input average values. Even though these approaches preserve brightness, they may not produce natural-looking images.

As a robust contrast enhancement method, Median-Mean-Based Sub-Image-Clipped HE (MMSICHE) [19] gives a controlled natural enhancement rate. This limited enhancement is performed in three phases. The mean and median brightness values is calculated firstly. Clipping the histogram using a plateau limit set as an occupied intensity median is done secondly. Bisecting the clipped histogram based on median intensity is processed thirdly to reach the final result before dividing the histogram into four sub images based on regional mean intensity.

Exposure-based Sub-Image HE (ESIHE) is designed for low exposure gray scale images [20]. Exposure thresholds of ESIHE are calculated to split the input image into sub images having different intensity levels. The histogram is also cropped using a criterion for controlling the enhancement rate. The criterion is a threshold value obtained from an average number of intensity occurrences. Each individual histogram of sub-images is separately equalized to be finally combined into one image.

Poddar et al. proposed a generalized HE method in the dynamic range of an input image without a parameter. The variants of this NMHE (Non-parametric modified HE) method both preserve and increase the brightness adaptively [21].

The brightness preserver RSWHE (Recursive Separated and Weighted HE) divides an image into sub-histograms recursively using a normalized power law function independently on each sub-histogram [22]. Similarly, Sim et al. present a method called RSIHE (Recursive Sub-Image HE) for gray-level images [23]. In same filed, RMSHE (Recursive Mean-Separate HE) is a scalable brightness preserver method [24].

Another computationally efficient framework is introduced in the wavelet domain. This method is capable of enhancing both local and global brightness and contrast likewise preserving color consistency [25]. Iqbal et al. presented a method to enhance medical image resolutions using dual-tree complex wavelet transform, singular value decomposition, and non-local means [26]. Wong et al. proposed a method which maximizes the coverage of permitted intensity ranges by employing an optimal weight factor strategy to combine both the input and interim images. The optimal weighting factor, which is found iteratively, enables optimum image enhancement while reducing artifacts [27].

Apart from these linear models, image contrast enhancement can be considered as an optimization problem using different types of approaches to reach an optimal solution [28]. Similarly, an image improvement method enhances the details in poorly illuminated areas using a bad illumination pass filter (BIPF) and an adaptive logarithmic transformation [29]. A survey of various methods is recently performed where the different types of HE algorithms are concisely explained in different aspects [30]. In this survey study [31], HE methods are almost classified into some categories. Local methods are listed as LHE, CLAHE, EHS, DHE, MMSICHE, ESIHE, and DSIHE. Adaptive ones can be AHE and CLAHE. BBHE, BPDFHE, and MMBEBHE are in the brightness preserving class. GHE and the proposed LDR-HE methods might be also categorized as global.

3 Proposed method: LDR-HE

In this study, a scalable and adaptable HE design is introduced by implementing Quantized Discrete Haar Wavelet Transform (Q-DHWT) on the probability mass function (PMF) of histograms. This section provides a brief background, the description of the Q-DHWT model, and the details of the suggested LDR-HE model.

3.1 Histogram equalization (HE)

Histogram indicates color distributions of an image. It also depicts the number of pixels in each of color ranges that span the set of all colors. Histogram equalization mainly boosts the overall contrast. Most common color intensities are equally dispersed on the histogram via this regulation to obtain higher contrast. This technique is very advantageous for images that have both a bright and dark background and foreground. Conventional HE techniques firstly transform the signals into a frequency domain to perform some manipulations. Then, the signals are simply restored by a reversible operation to get a processed image. However, throughout this process an increase in the existing background noises might occur while some necessary signals are reduced.

3.2 Haar wavelet transform (HWT)

Wavelets, which are an efficient tool for analyzing data, are mathematical functions developed to sort data by frequency. The wavelet transformation converts data from spatial domain to frequency domain, and it stores each component on the corresponding resolution scale. A wavelet represents an orthogonal basis of a vector space. The forward Haar transform might be essentially described as both averaging and differencing. The partial Haar function $\psi(t)$ for high pass and the scaling function $\varphi(t)$ for low pass are represented in Eq. 1.

$$
\begin{aligned}
&\psi(t) = \begin{cases} 1 & t \in [0, 1/2) \\ -1 & t \in [1/2, 1) \\ 0 & t \notin [0, 1) \end{cases} \\
&\psi_i^j(t) = \sqrt{2^j} \times \psi\left(2^j t - i\right), \\
&j = 0, 1, \dots \text{ and } i = 0, 1, \dots, \left(2^j - 1\right), \quad t \in \mathbb{R}, \{i, j\} \in \mathbb{Z} \\
&\varphi(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{1}
$$

For an input which are represented by a list of 2^n numbers, the HWT might be considered to plainly pairing up input values, saving the difference, and passing the sum. Recursively, this procedure repeats itself by pairing up the sums to afford the next scale: lastly resulting in 2^{n-1} differences and one final sum. Let X be a vector with N values (bins), $X = \left[x_1, x_2, \dots, x_N\right]$, where N should be the power of 2. The number of the total recursive steps (Ω) for a wavelet transform is obtained by $\log_2 N$. Then, the average of pair-wise numbers in the vector (s_k) for low pass and the directed distances of pair-wise numbers (d_k) for high pass are calculated in the Formula 2 as follows:

$$
\begin{aligned} s_k &= \frac{\left(x_{2k} + x_{2k+1}\right)}{2} \\ d_k &= \frac{\left(x_{2k} - x_{2k+1}\right)}{2} \end{aligned} \quad \text{for} \quad k = 0, \dots, \left(\frac{N}{2} - 1\right) \tag{2}
$$

The second data list $\boldsymbol{d}$ is obtained so that the original vector X can be restored from $\boldsymbol{d}$ and $\boldsymbol{s}$ as $X \rightarrow [s|d]$. The process is invertible since:

$$
x_{2k} = s_k + d_k = \frac{\left(x_{2k} + x_{2k+1}\right)}{2} + \frac{\left(x_{2k} - x_{2k+1}\right)}{2} \tag{3}
$$

$$
x_{2k+1} = s_k - d_k = \frac{\left(x_{2k} + x_{2k+1}\right)}{2} - \frac{\left(x_{2k} - x_{2k+1}\right)}{2} \tag{4}
$$

Finally, the vector in the inverse operation is decomposed and mapped as:

$$
\left[x_1, x_2, \dots, x_N\right] \rightarrow [s|d] = \left[s_1, \dots, s_{N/2} | d_1, \dots, d_{N/2}\right] \tag{5}
$$

3.3 LDR-HE (low dynamic range histogram equalization)

Figure 1 represents the overall structure of the proposed LDR-HE transformation model. An image is firstly transformed from spatial domain into frequency domain by Q-DHWT. The quantization part in the sub-band softens the high frequencies including natural noises. Namely, high-frequency amplitudes in the frequency domain are dumped in order to eliminate existent noises. Then the suppressed signals are inversed again in the reconstruction phase to get an image in higher contrast. At the final step, the original image and the processed image are compared in terms of a set of performance metrics.

In this study, an integration of Quantized Discrete HWT on 1D normalized histogram (PMF) of 2D image is presented. Since the histogram bins can contain a value that is a multiple of 2 (2^n where $n = 8$), HWT becomes applicable. Q-DHWT separates any signal into two bands: high pass and low pass sub-bands. The low pass band keeps the mean value of the inputs whereas the high pass band keeps the differences between the inputs. These differences indicate how the signal is changing in the multi-resolution framework of HWT. Knowing the noise generally exists in high dynamic range, the principle is to reduce the dynamic range of the input vector to a lower standard deviation. Hence, illumination variations and existing noises are suppressed so that image quality is enhanced while performing the procedures.

The LDR-HE transformation function $y = T(x)$ applies a particular Quantized Discrete HWT $\psi_{f(x)}$ on the quantized PMF (*Q-pmf*) and calculates the CDF as follows:

$$
Q\_pmf\left(x_i\right) = \begin{cases} \frac{x_i}{\alpha}, & \left|x_i\right| > \mu_{high\_freq} \\ x_i, & otherwise\left(\left|x_i\right| \le \mu_{high\_freq}\right) \end{cases} \tag{6}
$$

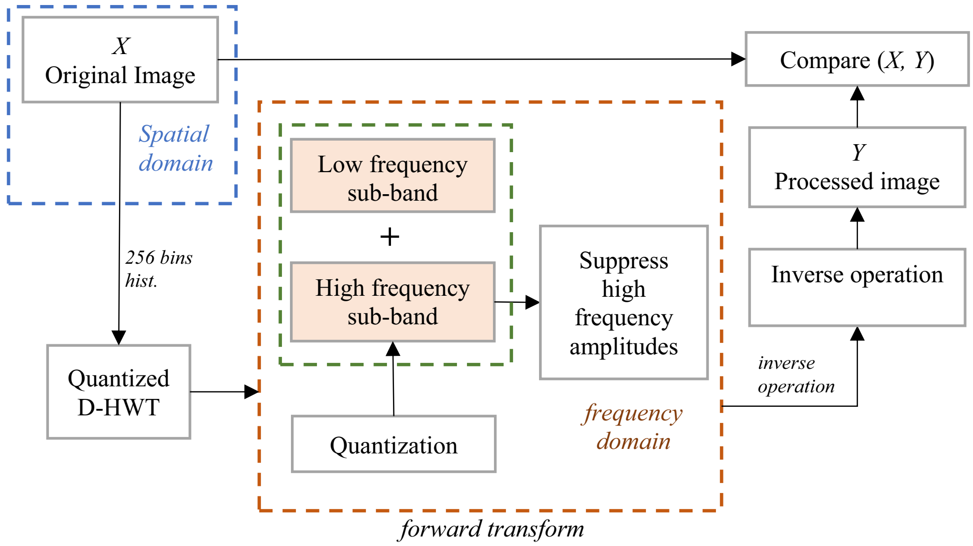

Fig. 1 LDR-HE flowchart

$$\alpha = c \times \frac{\sigma_{high\_freq}}{\mu_{high\_freq}} = c \times \frac{\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\left|x_i\right| - \mu_{high\_freq}\right)^2}}{\frac{1}{N}\sum_{i=1}^{N}\left|x_i\right|} \tag{7}$$

$$cdf_x(i) = P_x(i) \times \sum_{j=0}^{i} \psi_{Q\_pmf(x_i)}, \quad i \in [0, L-1] \tag{8}$$

where x_i is the the ith HWT coefficient in the high-pass band of an input image x. μ_{high_freq} and σ_{high_freq} are the mean value and the standard deviation of the absolute HWT coefficients, respectively. Actually μ_{high_Pass} is the threshold value for each x_i signals. In the HWT transform, the first coefficient bin is the mean value of the whole sequence. The cumulative distribution function $cdf_x(i)$ corresponding to p_x can be called an accumulated and normalized histogram. The quantized probability mass function $P_x(i)$ is the normalized probability in between [0, 1] for a pixel of level i. L is the total number of color intensities, generally 8 bits.

In Eq. 7, c is a constant coefficient, generally in between (1, 10] to empower the ratio of standard deviation and mean. In the experimental studies, c is tested from 1 to 100. The most suitable c is observed as 8 for the best performance. α is actually the coefficient of variation (CV) of the HWT signals. CV is simply the ratio of σ/μ.

As a new contribution, this $Q\_pmf\left(x_i\right)$ function plays a crucial role in the LDR transformation, since it softens the higher amplitudes. The amplitudes of the signals are derived from the absolute value of signals with $\left|x_i\right|$. If the signal amplitude x_i is greater than the threshold (μ_{high_freq}), it is divided by the alpha (α) parameter to reduce its strength. Alpha is the damping factor which decreases the amplitude of oscillations. In other words, it weakens the higher amplitudes of the frequencies in the high pass spectrum. In low dynamic range histogram equalization (LDR-HE), the value of α parameter can be in between $[1, +\infty)$ which represents the empowered strength of CV. If α is set to 1.0, the conventional HWT procedure solely is applied. The more α value there is, the more reductions of the amplitudes there are in the signal power.

Alternatively, the alpha parameter can also be defined as the signal-to-noise ratio (SNR) as described in Eq. 9 [32]. SNR is basically the inverse form of CV. Thus, in the *Q-pmf* function (Eq. 10), α should become a multiplier. In the experimental studies, exactly similar experimental results have been taken with SNR if the c constant is set to a real number in between (0, 1].

$$\alpha = SNR = c \times \frac{\mu_{high\_freq}}{\sigma_{high\_freq}} \tag{9}$$

$$Q\_pmf\left(x_i\right) = \begin{cases} x_i \times \alpha, & \left|x_i\right| > \mu_{high\_freq} \\ x_i, & otherwise\left(\left|x_i\right| \le \mu_{high\_freq}\right) \end{cases} \tag{10}$$

Instead of the color intensity histogram, a new array is created with the absolute values of the high pass coefficients in the high pass band because it is more logical to calculate the contrast based on edge magnitudes rather than color intensities. Increasing the contrast value ordinarily makes the edge lines appear clearer, since the edge information is kept in the high pass band.

If the absolute value (magnitude) of high frequency values is greater than the average (μ_{high_freq}) of the coefficients, it is divided by the CV value in order to increase the contrast of low-contrast images. This operation is performed for the frequencies which have greater amplitude values than the mean amplitude. Therefore, by reducing the high alterations in the frequencies, high dynamic range becomes low dynamic range.

In the inverse operation in HWT, since the mean value (the first value of the vector) in the low pass band is not manipulated by any means, a new histogram is created from the fixed (unchanging) coefficients with reduced frequency coefficients to preserve the total signal strength. It is very crucial to remain the mean value in the first index of the vector unchanged. Thus, this new processed histogram finally becomes a low dynamic range histogram.

When HE is done separately in each color channel, both the cumulative dynamic range and the overall contrast increase are acquired as well. If contrast stretching is performed from a lower range, the cumulative contrast is absolutely enhanced more than normal. As an example, when an 800×600 image is constructed from a 640×480 image, the quality definitely decreases whereas the contrast increases. On the other hand, when an 800×600 image is constructed from a 320×240 image, the quality decreases a lot and the contrast increases much more. This is exactly what it is done in LDR-HE by decreasing the dynamic range. A histogram equalization is created from the lower range. Thus, the contrast increases even more than the conventional methods. Furthermore, the quality of images does not deteriorate much since the dynamic range of the frequencies is reduced.

If the signal's absolute change is above the mean of high-pass values, then the corresponding histogram bin is updated with the alpha parameter in CV. The more alpha there is, the more dynamic range of histograms is reduced. With the help of this method, the frequencies with high changes in the histogram are dumped and the contrast is increased after the inverse operation. This suggested LDR-HE method stands as a novel technique to enhance the image quality while adjusting the dynamic range by filtering wavelet coefficients. The technique is applied to the *Q-pmf* which is the normalized version of the histogram. By this method, the PMF signals are boosted while the mean value of the PMF is being preserved. After obtaining the boosted PMF, CDF is constructed to achieve a better histogram equalization.

These manipulations of the high-frequency components make the edge lines more noticeable. This leads to the preservation of image quality. If the same operation of LDR is performed in the spatial domain, the quality would have been dramatically reduced, and the total time complexity will completely increase.

On the other hand, the standard deviation of an input image can dramatically decrease when a higher alpha parameter is set. Low standard deviation values indicate high uniform distributions, and it results contrast increases in the processed image. This reduction in standard deviation indicates the improvement in the image quality by stretching the bins in the frequency domain. This simple operation exactly provides a dynamic range reduction. It is important to emphasize that lower standard deviation indicates a uniform distribution, namely a contrast enhancement operation. Additionally, when the standard deviation decreases, the dynamic range also decreases.

LDR-HE apparently softens drastic and sudden changes in the color bins so that the details in the image can be visually observed by changing the standard deviation values. The entropy is also increased since the variety in color intensities is increased in order to create a uniform distribution.

$$\int_{0}^{L-1} x_i P(x_i) \mathrm{d}x \cong \int_{0}^{L-1} y_i P(y_i) \mathrm{d}y \tag{11}$$

where L is the color capacity. Equation 11 indicates that the summation of color bins for each input and processed images is almost correlated to each other. This facility of LDR-HE provides a framework which is trying to preserve the mean brightness. In practical applications, the division of the color bins with an alpha parameter results very sensitive floating point numbers. Since the outputs are rounded in calculations, the values of color intensities may also vary. This situation causes some negligible increase in brightness. As a result, LDR-HE apparently softens drastic changes in color bins so that the details in the image can be easily observed. In conclusion, this simple operation simply provides a dynamic range reduction.

3.4 Time and space complexities of LDR-HE

The time complexity of LDR-HE in Big-O notation is as given in the expanded formula 12 as:

$$O\left(I + X + \log_2 N + X + \log_2 N + I\right) \tag{12}$$

where, respectively, I is the acquisition cost for a gray level image, X is the construction cost for the 1D HWT bins (generally 2^8), $\log_2 N$ is the cost of recursive steps for the forward HWT operation phase, X is the quantized PMF operation, $\log_2 N$ is reconstruction phase of HWT, and finally I is the creation of output image. The summarized complexity is as $O\left(I + X + \log_2 N\right)$ since the coefficients are negligible in the complexity theory. The space complexity also is only as $O(X)$ because of the 1D vector for HWT bins. As there are very tiny complexity components, the computational time of LDR-HE becomes very small when compared with the others.

4 Experimental results and discussion

LDR-HE is implemented in Java, and comparative performance studies are conducted in the MATLAB 2021a environment by using a computer with normal specifications. Basically, all the experiments have been performed on the CSIQ benchmarking dataset [33] consisting of 30 RGB images in 512×512 resolutions with different distortion levels. Some of the images have low/average/high contrasts. Some parts of an image also have purposely low/average/high contrasts.

4.1 Image quality measurement criterions

Evaluation of image quality occurs when it is necessary to measure the fidelity of the encoded copy of the image compared with the original version. Quality assessments provide objective quantitative measurements. For this study, six different types of fundamental and modern assessment metrics are preferred to measure the performance and accuracy of the methods. Apart from the metrics described below, some other well-known metrics have been previously implemented and experimental results have been taken. The full extended outputs can also be received from the website [34] for examinations. The excluded metrics are SSIM (Structural Similarity Index), PCC (Pearson 2-D correlation coefficient), PCQI (Patch-based Contrast Quality Index), and MI (Mutual Index). Since these tools are not directly related to the HE assessments, they are not included into this study. In HE quality assessment, looking at one or two metrics alone is clearly leads to misinterpretations. Therefore, a particular set of metrics should be evaluated together for a better decision.

4.1.1 Mean squared error (MSE) and peak signal-to-noise ratio (PSNR)

MSE and PSNR are the well-known criterions used to evaluate the performance of manipulations. MSE shows the mean of the cumulative square error between filtered and original images [35]. Higher MSE value indicates the existence of noises in the image. PSNR is simply the logarithmic inverse of MSE which shows the measure of the peak error [36]. Obviously, the maximum similarity is achieved when the MSE is equal to zero. PSNR in decibels units calculates the peak signal-to-noise ratio between input and output images. Shortly, lower MSE and higher PSNR are better [37].

4.1.2 Quality-aware relative contrast measure (QRCM)

Another novel and modern metric for quality assessments in HE operations is QRCM [38]. It considers both the distortions resulting from the enhancement process and the level of relative contrast enhancement between test and processed images. This measure produces a real number in between [-1.0, 1.0] where 1.0 refers to a full improvement and -1.0 refers to a full level of contrast degradation. This measurement employs the gradient magnitudes of input and output images in order to calculate image quality degradation and the relative contrast change. QRCM focuses of contrast changes when there is an important difference between the gradients of input and output images. QRCM both measures the relative change of contrast and the distortion introduced on the output image relative to the input image. As a result, a value of QRCM close to zero is better.

4.1.3 Universal image quality index (UIQ)

UIQ is designed by modeling a combination of three components: loss of correlation, contrast distortion, and luminance distortion. In the Eq. 13, the UIQ value is the product of three components [39].

$$\mathrm{UIQ}(x,y) = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \cdot \frac{2\overline{xy}}{(\overline{x})^2 + (\overline{y})^2} \cdot \frac{2\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \tag{13}$$

where x and y are original and reconstructed images, respectively. $\overline{x}$ and $\overline{y}$ are the means of the images. σ_x and σ_y are the standard deviations. The first component is about the linear correlation coefficient between x and y, in the range of $[-1, +1]$. The second component calculates how close the mean luminance is between original and reconstructed images. This component is in the value range of [0, 1]. The third component, which is in the range of [0, 1], measures how similar the contrasts of the images are. This quality metric forms a combination of these three factors: luminance distortion, contrast distortion, and loss of correlation. Consequently, the UIQ value is between -1.0 and $+1.0$ where the highest value occurs only when original and enhanced images are identical.

4.1.4 Contrast Improvement Index (CII)

Contrast of an image might be basically explained as the difference between maximum and minimum pixel intensity. Particularly, it is the difference in luminance or color for a group of objects within the same field of view. In this study, edge-based contrast measure (EBCM) is preferred [40]. Since this metric is based on the fact that an improved image naturally has more edge pixels than the input image, EBCM determines the intensity of edge pixels. The contrast improvement ratio can be found $\mathrm{CII} = \mathrm{EBCM}(Y)$ / $\mathrm{EBCM}(X)$. It is true that lower and much higher ratios are obviously worse. However, a little increase in CII is better while keeping MSE and AMBE values smaller.

4.1.5 Absolute mean brightness error (AMBE)

AMBE measures the difference of intensity value between input and output images as seen in formula 14:

$$AMBE = \left| \frac{1}{NM} \sum_{i}^{N} \sum_{j}^{M} X(i,j) - \frac{1}{NM} \sum_{i}^{N} \sum_{j}^{M} Y(i,j) \right| \tag{14}$$

where N and M are the image sizes. A zero AMBE value denotes full brightness preservation. Slightly higher AMBE might be accepted as satisfactory. If AMBE is very high, it is apparently inconvenient.

4.2 Quantitative analysis of alpha parameter in LDR-HE

All tests are run on the same benchmarking dataset to compare with each other using 6 number of image quality metrics such as PSNR, MSE, QRCM, UIQ, CII, and AMBE. In a HE process, all the metrics but MSE should give higher values for better performance. But, AMBE should be closer to 0 since higher brightness damages the intensity values. Unfortunately, there is not only one metric for successful quality assessments in histogram equalizations and contrast enhancements. This suit of metrics should be examined together in order to observe how the performance is correlated with the coefficient parameter, c.

In the experimental studies, it is observed that the outputs of the quality metrics for both gray scale and RGB are almost correlated to each other. Thus, RGB outputs are represented in this paper since its luminance gives higher visual perception.

As a damping factor, the alpha parameter requires an appropriate value of the constant coefficient (c) in the calculation of σ/μ (coefficient of variation) to perceive the best enhancement. To find out the appropriate coefficient, a set of experimental procedure, starting from 1.0 to 100.0, is conducted as it is seen in Table 1. The coefficient is increased sequentially for assessment. The system is also tested with the high c values such as 10, 20, 30, 40, 50, and 100. These experimental results are taken over the 30 test images. In each cell of Table 1, the first value is the mean and after the sign ± the second value is the standard deviation.

The main purpose in HE operations is to increase contrast as possible as without damaging the intensities. The contrast improvement is analyzed by the PSNR, CII, and QRCM metrics. The brightness gain is calculated with AMBE. The similarity between input and output is checked with UIQ. The error rate in the processed image is measured with MSE. By checking these metrics together, a feasible and accurate assessment can only be made.

According to the results of Table 1, as the α value increases, both dynamic range and the gain in standard deviation decreases. Therefore, the output of the system resembles the original image since the α weakens the amplitude of signals by blocking the histogram synchronization.

MSE and contrast have a negative logarithmic correlation with the alpha parameter, when there is a sequential increment on the c parameter. UIQ roughly quantifies the proximity between input and output images like the SSIM and MI metrics. In case of higher c values, the UIQ converges to

Table 1 The LDR-HE performances according to the coefficient (c) of alpha (α)

Coeff. c	PSNR (higher is better)	MSE (lower is better)	QRCM (close to 0 is better)	UIQ (higher is better)	CII (slightly higher is better)	AMBE (lower is better)
1	13.8 ± 4.9	4895.8 ± 5892	− 0.217 ± 0.1	0.72 ± 0.22	2.97 ± 4.28	39.79 ± 38.9
2	19.6 ± 5.0	1365.9 ± 1812	− 0.105 ± 0.1	0.83 ± 0.20	1.99 ± 2.01	20.82 ± 20.8
3	23.0 ± 4.9	614.9 ± 811	− 0.067 ± 0.1	0.87 ± 0.18	1.66 ± 1.36	14.11 ± 13.8
4	25.4 ± 4.8	350.9 ± 459	− 0.048 ± 0.0	0.88 ± 0.17	1.46 ± 0.87	10.78 ± 10.4
5	27.2 ± 4.7	228.5 ± 296	− 0.038 ± 0.0	0.89 ± 0.16	1.31 ± 0.50	8.78 ± 8.3
6	28.6 ± 4.7	161.8 ± 209	− 0.031 ± 0.0	0.91 ± 0.15	1.23 ± 0.34	7.46 ± 7.0
7	29.8 ± 4.6	120.9 ± 154	− 0.026 ± 0.0	0.91 ± 0.14	1.19 ± 0.27	6.50 ± 6.0
8	**30.8 ± 4.5**	**94.1 ± 119**	**− 0.023 ± 0.0**	**0.92 ± 0.14**	**1.17 ± 0.23**	**5.78 ± 5.2**
9	31.7 ± 4.4	76.0 ± 94	− 0.020 ± 0.0	0.92 ± 0.13	1.15 ± 0.20	5.22 ± 4.7
10	32.4 ± 4.3	62.6 ± 77	− 0.018 ± 0.0	0.92 ± 0.13	1.13 ± 0.18	4.77 ± 4.2
20	36.9 ± 3.9	19.9 ± 21	− 0.009 ± 0.0	0.94 ± 0.10	1.06 ± 0.08	2.79 ± 2.2
30	39.1 ± 3.9	11.4 ± 10	− 0.006 ± 0.0	0.95 ± 0.08	1.04 ± 0.05	2.14 ± 1.6
40	40.4 ± 3.9	8.4 ± 7	− 0.005 ± 0.0	0.96 ± 0.08	1.03 ± 0.04	1.83 ± 1.3
50	41.2 ± 3.8	6.8 ± 5	− 0.004 ± 0.0	0.96 ± 0.06	1.03 ± 0.03	1.64 ± 1.1
100	43.2 ± 4.1	4.7 ± 5	− 0.002 ± 0.0	0.97 ± 0.05	1.02 ± 0.02	1.29 ± 0.9

nearby one and both QRCM and MSE converges to nearby zero.

These results indicate the strong similarity level between the original and processed images. For instance, if c is set to a very high value, e.g. 100, there is a slight change in the output image. Moreover, the contrast remains almost the same and there is a meagre brightness acquisition. As a result, the high value of c produces very low MSE results; however, it decreases the CII and AMBE rates at the same time.

It is noticed in empirical applications that both the negative values and real numbers in between [0, 1) for the coefficient c produces annoying artifacts in the processed image. Similarly, if the coefficient c is set to 1, the result of the α parameter becomes smaller than 1.0 since the formula $\alpha = c \times \sigma / \mu$. Consequently, if the α is smaller than 1.0, it does not damp the magnitudes of the high frequency signals. It should be emphasized that if the α is equalized to 1.0 with an appropriate c coefficient, the convolutional HWT without any extra operation becomes applicable. In each test image, if the α is equalized to 1.0 with different c values, it can be accepted as the initial reference state of HWT.

In Table 1, it is observed that if the c is set to a small value, excessive contrast improvement (CII) occurs with high MSE rates. Higher CII values are not preferable because of produced annoying artifacts. The system gives more satisfactory results when c is gradually increased. The appropriate c value in fact can be decided according to the needs. The balanced c value should be defined by checking the results of metrics. Actually, this embedded facility enables an adjustable and scalable HE mechanism.

This table also indicates that image quality is increased and the amount of noise within the dynamic range is reduced while increasing the coefficient c. In order to find the contrast-saturated level of c, in the experiments c is increased gradually. It is observed that there are no more concrete changes after $c = 8$. The best saturated c value to compete with the other well-known methods is figured out as 8 when the outputs are checked.

LDR-HE is also very sensitive to its c parameter. Since LDR-HE performs histogram equalization in low dynamic range depending on the parameter, the contrast value of pictures logarithmically decreases when the c value is increased. On the other hand, PSNR values increase logarithmically while MSE values decrease. It proves that image quality and structural similarity are increased by the amount of coefficient.

The positive or negative logarithmic correlations with alpha are caused by Quantized Discrete HWT in which the contrast (standard deviation) in the picture is decreased by the amount of alpha coefficient while preserving the mean brightness since HWT keeps the mean value of any input vector constant throughout the operations.

Furthermore, HE enhances the image quality by boosting the contrast in a low dynamic range in which noise elements do not generally exist. By this method, the histogram is equalized with less amount of noise and better results are obtained with respect to the other existing methods.

4.3 Qualitative analysis of alpha parameter in LDR-HE

Since the resolution of the test images is very big for visual comparisons, a special section is cropped from the *family* and *roping* images in Figs. 2 and 3, respectively, to assess the details easily. One of these values {1, 2, 3, 5, 8, 50, 10} is assigned to the c factor in order to analyze its effects both qualitatively and quantitatively.

First of all, when the coefficient c is set to 1, it gives an unsuccessful result with the highest MSE value when compared with the others. When $c = 1$, some visual distortions inevitably appear since the alpha is equalized a smaller value than 1.0. In these cases, the high frequency signals cannot be suppressed.

In the original image in Fig. 2, the six geometrical light rays around the sun are simply noticeable. When the c is set to 1, 2, 3, or 5, the six ray beams are intertwined. It is apparent that image information is totally lost around the sun. Besides, there are some excessive numerical results of MSE, CII, and AMBE. Additionally, when the c is set less than or equal to 5, these annoying cases occur in the image. Firstly, some improved noises over the falling lights onto the sea are generated. Secondly, the actual color intensities of the family image are strongly manipulated. Lastly, the geometrical segments in the sky also are overlapped. This situation is obviously accepted as annoying artifact.

However, when c is set to 8, it becomes very easy to notice the light beams separately. At this stage, the contrast gain (CII) is 1.56 with a low error rate (MSE = 339). For this specific test case, the balanced c factor should be 8. The foreground can be easily and apparently extracted from the background in this image after the histogram equalization process.

In contrast, when $c = 100$, the minimum MSE, CII and AMBE values and the maximum PSNR, QRCM, UIQ values are received. The numerical results indicate a statement that when the coefficient is set to 100, the original and processed images becomes almost identical. Greater c coefficient values slightly increase the PSNR value; in fact, it does not make any improvement in the contrast at all. When $c = 8$, a balanced output is received.

Since the cropped section from the *roping* image has very dark and very bright sections, it is not easy to process. In Fig. 3, enormous contrast enhancement and over-luminance can be seen both visually and numerically when the c is set to less than or equal to 5. Also, the AMBE and MSE values

Original	c=1	c=2	c=3	c=5	c=8	c=50	c=100
	PSNR:5.33 MSE:19050 QRCM: -0.264 UIQ:0.391 CII:71.57 AMBE:134.0	PSNR:11.0 MSE:5172 QRCM:-0.164 UIQ:0.551 CII:28.92 AMBE:69.6	PSNR:14.5 MSE:2313 QRCM:-0.089 UIQ:0.642 CII:19.50 AMBE:46.6	PSNR:18.8 MSE:850 QRCM:-0.039 UIQ:0.734 CII:3.03 AMBE:28.2	PSNR:22.8 MSE:339 QRCM:-0.018 UIQ:0.795 CII:1.56 AMBE:17.8	PSNR:37.4 MSE:12 QRCM:-0.001 UIQ:0.810 CII:1.09 AMBE:3.2	PSNR:40.8 MSE:5 QRCM:0.001 UIQ:0.907 CII:1.06 AMBE:2.1

Fig. 2 Cropped sections from the *family* image

Original	c=1	c=2	c=3	c=5	c=8	c=50	c=100
	PSNR:7.0 MSE:13012 QRCM:-0.402 UIQ:0.269 CII:3.797 AMBE:91.6	PSNR:12.6 MSE:3593 QRCM:-0.086 UIQ:0.358 CII:2.087 AMBE:47.0	PSNR:16.1 MSE:1611 QRCM:-0.028 UIQ:0.413 CII:1.629 AMBE:31.5	PSNR:20.4 MSE:593 QRCM:-0.005 UIQ:0.473 CII:1.281 AMBE:19.2	PSNR:24.3 MSE:241 QRCM:0.005 UIQ:0.542 CII:1.153 AMBE:12.3	PSNR:38.1 MSE:10 QRCM:0.004 UIQ:0.804 CII:1.029 AMBE:2.6	PSNR:42.1 MSE:4 QRCM:0.003 UIQ:0.840 CII:1.017 AMBE:1.7

Fig. 3 Cropped sections from the *roping* images

are noticeably very high. The patterns in the striped shirt can be seen when the c is set to 5 or more. Again, the processed image visibly looks very similar to the original in higher c parameters. Correspondingly, the ideal coefficient should be set to 8 for the roping image since the highest QRCM value is received at this level. In these *family* and *roping* test images, noises can be visually and numerically detected in the images where there are higher MSE results.

4.4 Quantitative performance comparisons

The most important and most cited HE techniques are compared with the proposed one in terms of different types of quality metrics. An extensive benchmarking study is performed by these methods:

1. LDR-HE (Low Dynamic Range HE)
2. HISTEQ (Global HE or Conventional HE)
3. CLAHE (Contrast Limited Adaptive HE) [12]
4. LHE (Local HE)
5. MMSICHE (Median-Mean-Based Sub-Image-Clipped HE) [19]
6. BPDFHE (Brightness preserving dynamic fuzzy HE) [15]
7. EHS (Exact histogram specification) [13]
8. ESIHE (Exposure-based Sub-Image HE) [20]
9. BBHE (Mean preserving Bi-HE) [16]

10. MMBEBHE (Minimum Mean Brightness Error Bi-HE) [18]
11. DSIHE (Dualistic Sub-Image HE) [17]
12. RMSHE (Recursive Mean-Separate HE) [24]
13. RSIHE (Recursive Sub-Image HE) [23]
14. RSWHE (Recursive Separated and Weighted HE) [22]
15. NMHE (Non-parametric modified HE) [21]

The achievements of the presented method can be compared by analyzing the results in Table 2. The first values are the means over the dataset. The real values after the ± sign are the standard deviations. Lower standard deviation is better. The best performances in each column are marked in bold for a global comparison. The algorithmic structure of the methods in the comparison list has been explained in the literature section. Since the HE methods are in the categories of global, local, adaptive or recursive, the test results might differ from each other.

According to results in Table 2, LDR-HE overwhelms the existing methods in terms of the quality metrics. According to the UIQ criterion, LDR-HE keeps the structural similarity with respect to original images much more than the others. Additionally, the MSE for LDR-HE is the smallest of all. The performance of the HE methods is also measured with PSNR where the proposed method overwhelms when compared with the existing methods. When the contrast achievement rate (CII) is around 1.17, LDR-HE method attains 94.1 MSE rate while BPDFHE reaches 112.3 MSE rate.

There is a strong correlation between AMBE and MSE error rates. If one of them is high, the other is high also. Thus, this relationship indicates that brightness improvement produces some errors in the color intensities.

Furthermore, there is a noise factor in the test images. These noises are also enhanced during most of the HE methods. Therefore, the CII and PSNR metrics cannot be solely employed in comparisons, the MSE, QRCM, and UIQ metrics should be considered together. Some of the methods have some drawbacks such as over increase in contrast while producing more MSE rates. These HE methods increase MSE values uncontrollably by stretching of histograms bins equally by force over the range. LDR-HE with a user-defined parameter can adjust the enhancement degree by a potentiometer (regulator), the alpha parameter.

LDR-HE is trying to suppress the brightness gain also. When compared with the other methods, LDR-HE is the fourth one which keeps the brightness gain minimum. In an ascending order, the AMBE values of these methods BPDFHE, MMSICHE, and ESIHE are 0.04, 3.12, 5.29, respectively. However, the overall contrast improvement and minimum error rates in terms of QRCM and MSE are lower than LDR-HE's. Besides, the structural similarity performance of LDR-HE in terms of UIQ is not much lower than MMSICHE and BPDFHE.

4.5 Qualitative and visual performance comparisons

Figures 4, 5, and 6 objectively and subjectively present the whole performances of the methods. All the processed images in their original sizes by these algorithms can be also downloaded from the website [34] for examination since

Table 2 Total experimental comparative results

	PSNR (higher is better)	MSE (lower is better)	QRCM (close to 0 is better)	UIQ (higher is better)	CII (slightly higher is better)	AMBE (lower is better)
LDR-HE	**30.81 ± 4.5**	**94.1 ± 119**	**− 0.023 ± 0.0**	0.92 ± 0.14	1.17 ± 0.23	5.78 ± 5.23
CLAHE	16.97 ± 2.2	1454.9 ± 618	− 0.264 ± 0.1	0.81 ± 0.12	1.62 ± 0.96	15.94 ± 10.47
HISTEQ	15.82 ± 4.2	2489.6 ± 2137	− 0.183 ± 0.1	0.76 ± 0.19	2.22 ± 2.27	28.16 ± 22.74
BPDFHE	30.09 ± 5.2	112.3 ± 115	− 0.058 ± 0.1	0.94 ± 0.07	1.04 ± 0.18	**0.04 ± 0.04**
EHS	15.96 ± 4.1	2374.2 ± 1991	− 0.197 ± 0.1	0.77 ± 0.19	2.22 ± 2.27	27.76 ± 22.21
ESIHE	26.50 ± 5.1	277.9 ± 386	− 0.072 ± 0.1	0.92 ± 0.10	1.47 ± 1.27	5.29 ± 5.49
LHE	10.27 ± 2.3	7087.4 ± 4446	− 0.442 ± 0.1	0.70 ± 0.15	**2.67 ± 3.24**	37.31 ± 30.63
MMSICHE	28.68 ± 3.8	131.1 ± 143	− 0.066 ± 0.1	**0.95 ± 0.06**	1.14 ± 0.38	3.12 ± 3.19
BBHE	22.17 ± 5.7	686.9 ± 565	− 0.134 ± 0.1	0.87 ± 0.14	1.38 ± 0.72	13.60 ± 10.85
MMBEBHE	19.17 ± 5.0	1267.9 ± 1093	− 0.165 ± 0.1	0.85 ± 0.15	1.88 ± 1.48	21.11 ± 12.90
DSIHE	21.83 ± 5.8	746.8 ± 621	− 0.139 ± 0.1	0.87 ± 0.14	1.41 ± 0.80	14.41 ± 11.39
RMSHE	19.81 ± 2.9	833.8 ± 614	− 0.036 ± 0.1	0.84 ± 0.08	1.12 ± 0.84	18.57 ± 11.58
RSIHE	19.57 ± 2.9	883.8 ± 618	− 0.046 ± 0.1	0.86 ± 0.07	1.17 ± 0.85	19.57 ± 11.45
RSWHE	21.21 ± 5.9	963.5 ± 1021	− 0.122 ± 0.1	0.88 ± 0.08	1.45 ± 0.89	17.95 ± 16.13
NMHE	25.90 ± 6.2	333.7 ± 414	− 0.066 ± 0.0	0.92 ± 0.11	1.61 ± 1.30	10.24 ± 7.95

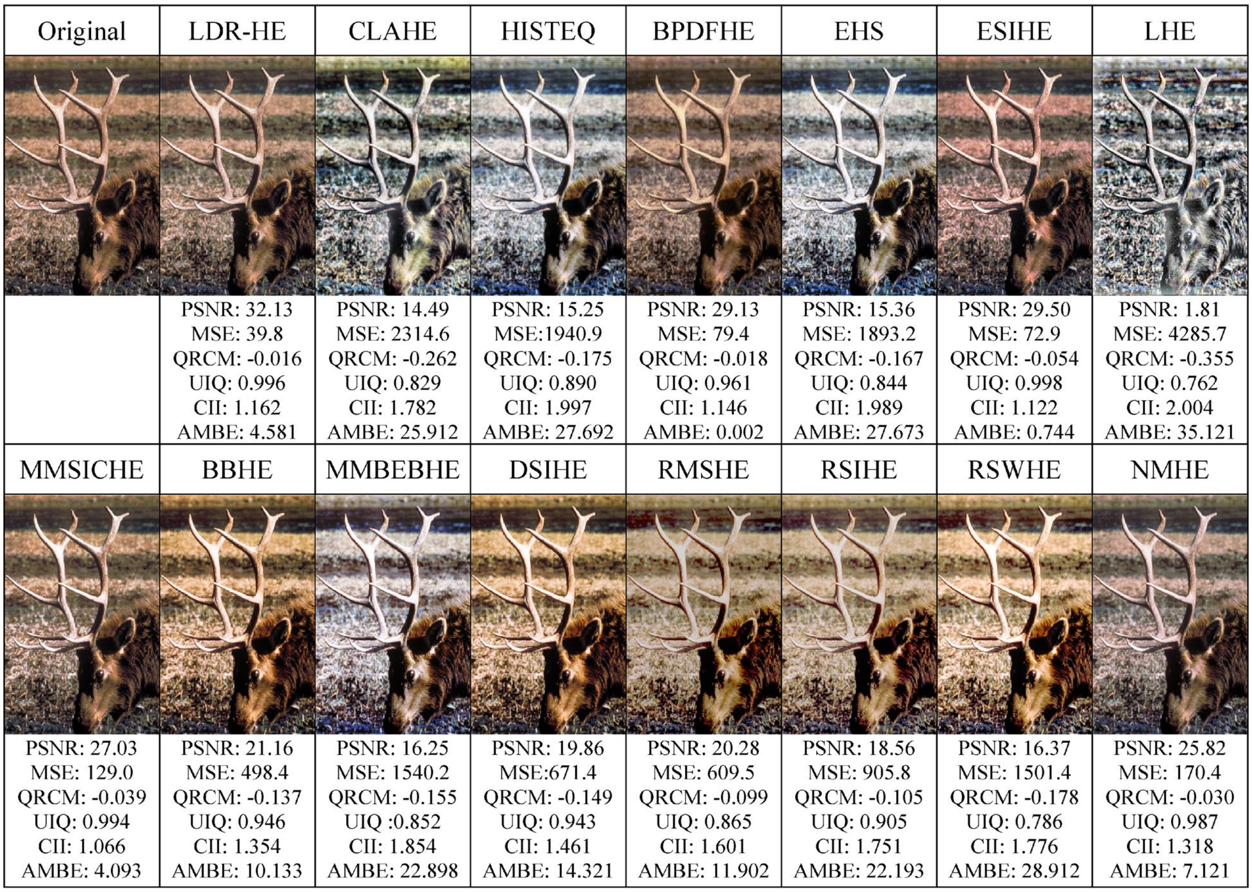

Fig. 4 Sample experimental results of HE methods over the *elk* image

some image information and details in these tables might disappear in resized forms.

Figure 4 presents the snapshots and the full test results of the *elk* image. It is also low-contrasted image. If the MSE rates is obviously bigger, unpleasant artifacts even in the resized images can be clearly seen. In those images, there are very low UIQ and PSNR values. Likewise, there are greater AMBE and CII values. This collection of the results quantitatively signifies the worst process where there are unwanted noise amplifications and over intensity saturations.

In the *elk* images of HISTEQ, CLAHE, EHS, LHE, and MMBEBHE, it is observed that there is an over-enhancement problem. These perceiving contrasts produce annoying artifacts which can be visually seen in the images. Furthermore, stimulated amplification of noise exist in these perceived images. For instance, RSWHE shows better contrast enhancement with 1.776 CII and 16.37 PSNR values. But it gives poor brightness preserving with the 28.912 AMBE value. Some multi-HE methods like RMSHE, RSIHE, and RSWHE have given almost similar brightness and contrast enhancements since their algorithmic structure are similar to each other. They all recursively separate the histogram using median and mean in sub-histograms. The recursively iteration number is set to 1 in the experiments.

BBHE is a clipped HE method and it decomposes an image into sub-images using mean brightness error. DSIHE and MMBEBHE are in the same categorization. Actually, MMBEBHE is an extended version of BBHE. Therefore, MMBEBHE produces more CII rates. But, the AMBE and MSE rates are two or three times greater. This situation indicating a full gain without any loss cannot be possible. Besides, the quantitative results of BBHE and DSIHE are very correlated to each other. Likewise, the processed images are almost similar when zoomed.

BPDFHE partition the global histogram bin into multiple sub-histogram bins based on the local maxima. Hence, it is clearly a brightness preserving method with keeping the originality with −0.018 QRCM and 0.961 UIQ values. However, the contrast gain is limited (CII = 1.146, PSNR = 29.13) and the error rate (MSE = 79.4) is greater than LDR-HE's (MSE = 39.8). Also, LDR-HE has higher contrast achievements (CII = 1.162, PSNR = 32.13) with negligible brightness improvement with a 4.581 AMBE rate. As it seen, this model has tried to reduce the brightness gain.

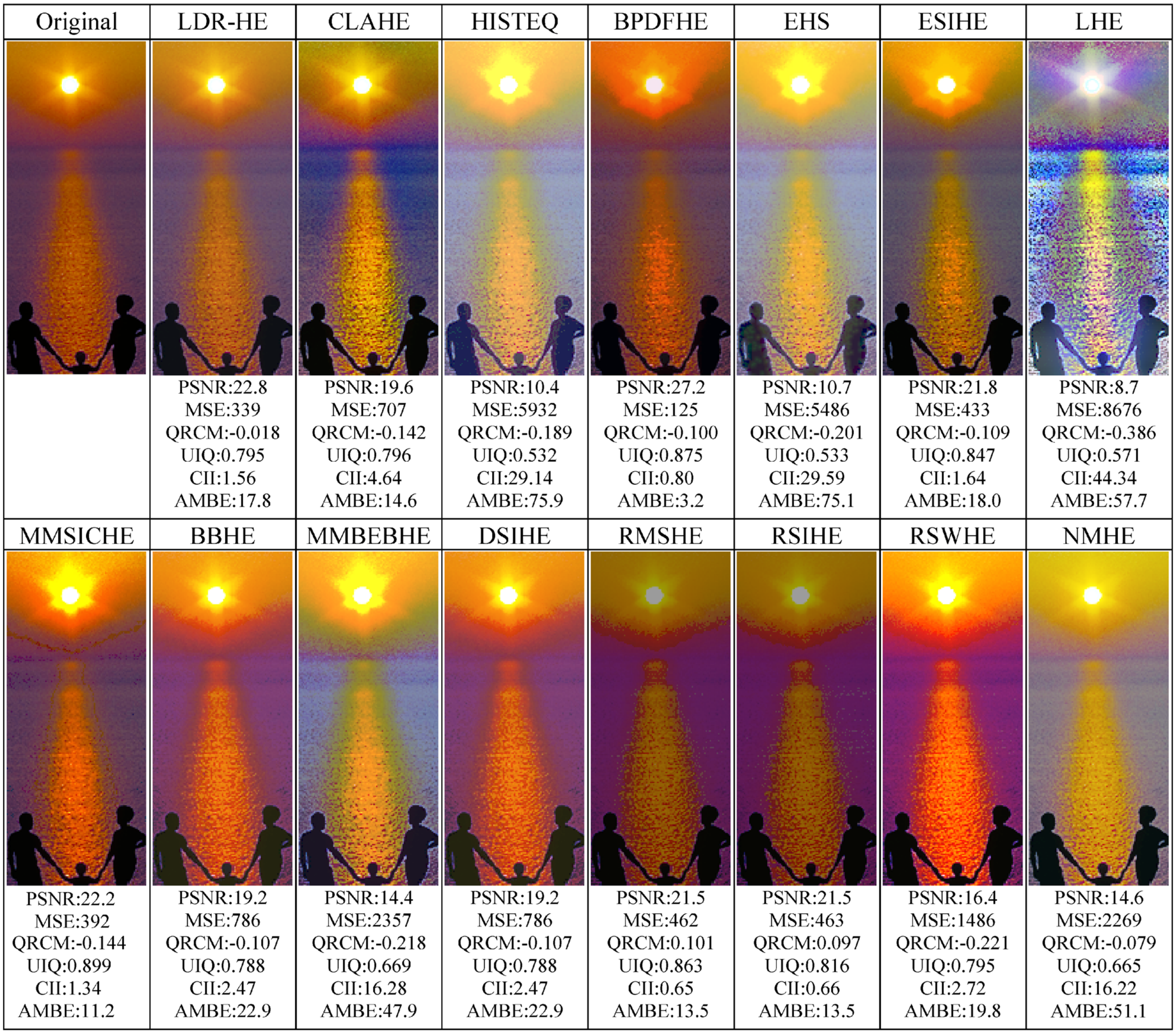

Fig. 5 Cropped sections from the *family* image

In Fig. 5, a special section including a family in the sunset is cropped from the original *family* image. This figure is almost the summary of the study by presenting both visual and numerical satisfactory results. In the cropped images, the unwanted artifacts around the sun can be easily recognized in most of the methods. Besides, the color intensities are mostly manipulated in the *family* images by the HISTEQ, BPDFHE, EHS, ESIHE, LHE, RSWHE, NMHE, and MMBEBHE methods. Most of the sky in the images is not smooth as in the original. Furthermore, foreground and background in most of the results are intersected. It is true that the six light beams around the sun in the input image are in a certain shape, structure and pattern. However, the sun and its surrounding lights are distorted in most of the methods except LDR-HE, CLAHE, and NMHE. When these three methods are compared among them, the ideal numerical MSE, QRCM, AMBE results are taken without losing any image information. RMSHE and RSIHE produce the greatest QRCM results of all. However, there is a dramatic decrease in the contrast improvement (CII is lower than 1.0). It is definitely against the philosophy of HE. The third successful QRCM gain is received with LDR-HE where a satisfactory contrast gains with the 1.56 CII. These measurements demonstrate the successful accomplishments of LDR-HE.

In Fig. 5, the PSNR value of the proposed method is lower than BPDFHE's. Even though the BPDFHE is quantitatively is better than LDR-HE, its color distribution, light beams, and sky are not pleasant when compared with LDR-HE's. In terms of PSNR, MSE, QRCM, UIQ, and AMBE, the most successful one is seemingly the LDR-HE method. However, in terms of AMBE, LDR-HE is not better than

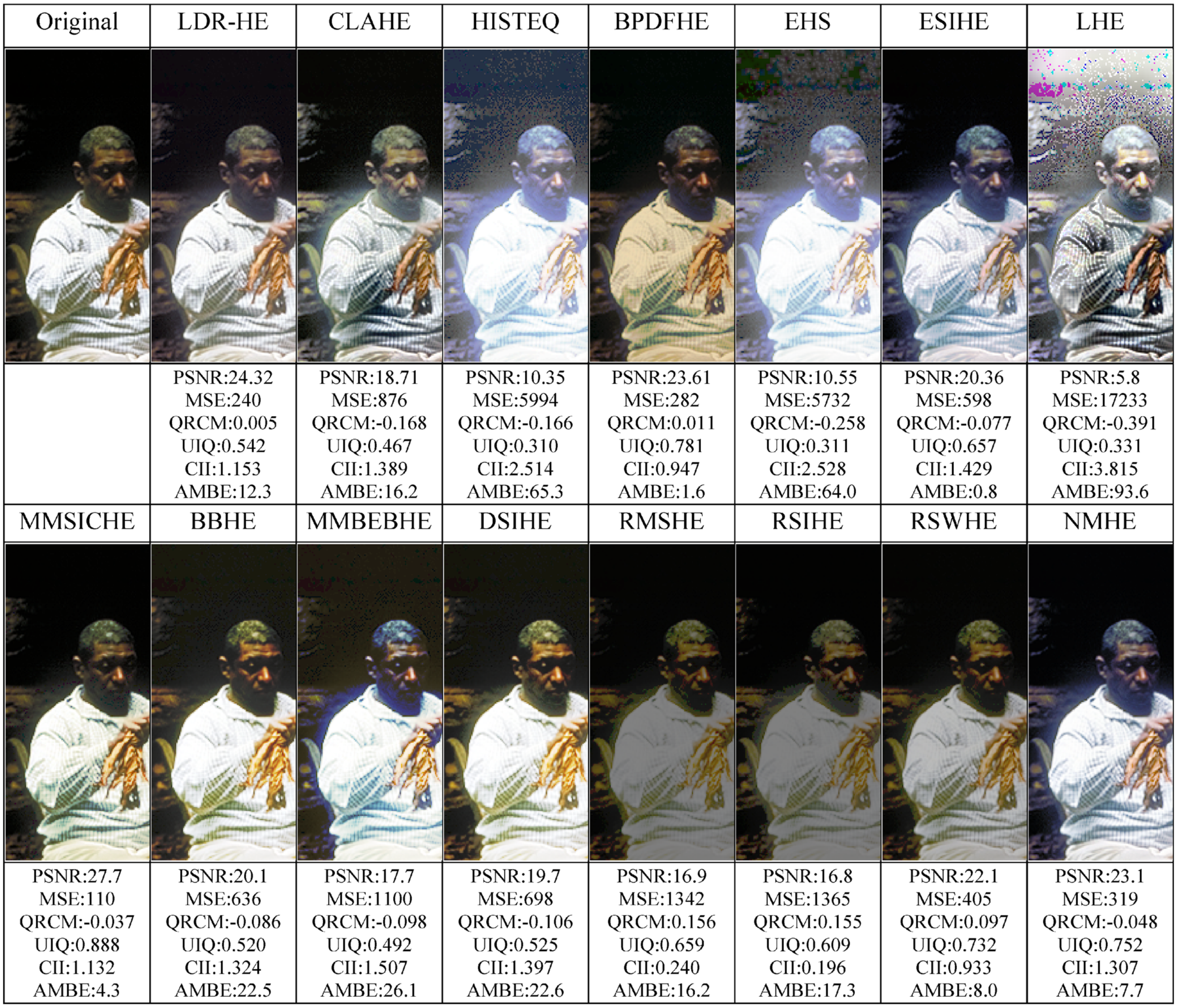

Fig. 6 The cropped *roping* image

CLAHE. CLAHE creates less brightness in the processed image. But, if these two algorithms are compared in terms of the most important metrics (PSNR, MSE, and QRCIM), the proposed method is definitely considered better.

In the output image of NMHE, the color intensities are strongly manipulated. The orange colors are transformed into almost yellow. In CLAHE, at the horizon, the purple color is transformed into blue. In CLAHE, the sky has also annoying artifacts and unwanted noises.

Unnatural appearances and annoying artifacts in Fig. 6 can be visually analyzed. Besides, the selected assessment metrics can give satisfactory results to some extent. Figure 6 also demonstrates the resized snapshots of the *roping* image in terms of six metrics. *roping* is a low-contrast image and not easy to equalize since there are sharp peaks in the histograms, e.g., very dark and very bright colors together in one image. Additionally, normalizing the peaks is very hard for these methods. Hence, lower contrast occurs in most of the methods such as BPDFHE, RMSHE, RSIHE, RSWHE. They do not perform well in terms of the CII criterion. However, high contrasts in the processed images of CLAHE, HISTEQ, EHS, and LHE produce unnatural annoying artifacts. These methods over-brighten the *roping* image. Therefore, the cumulative contrast achievement is accepted as lower. The color distribution in each algorithm is different. Even though RMSHE and RSIHE have better QRCM values, they cannot present an equalized distribution in histogram.

When the experimental results in Fig. 6 is examined quantitatively, the best performance (the greatest QRCM) is taken with the RMSHE and RSIHE methods. However, there are dramatic losses in contrast when the CII results are examined. LDR-HE gives a balanced output with the numeric test results. The successful results of LDR-HE can be also visually examined. This actually indicates that the human visual system is more contrast-sensitive than quality metrics.

In the experimental results of the *roping* image, only the PSNR result of MMSICHE is bigger than the LDR-HE's. Also, the MSE of MMSICHE is the lowest one. In this special example, it seems that MMSICHE overwhelms LDR-HE. However, when the QRCM and CII results are compared between them, the overall HE performance is better in LDR-HE.

4.6 Discussions

Local, global, adaptive, and recursive HE models are presented together in this study. In fact, it is not convenient to compare different types of HE methods in one study. Each HE method should be categorically compared with its relative methods. Also, there are some brightness preserving methods which should be handled in a different category. Some of the HE methods prevent over amplification while others do not. However, all the methods in one basket are evaluated in order to demonstrate the overwhelming performance of LDR-HE.

It is a general fact that if there is a gain from a system, there should be an extra loss at the same time. In contrast, in this proposed model, the high-performance results similarly depend on an appropriate value of the alpha parameter. In addition, this presented scalable system distributes histograms equally in low dynamic range by dumping the high frequencies where there are possible noises.

Most image contrast enhancement methods, as experienced in this study, generate undesired results. In the convolutional methods, existent noises are also stretched in HE operations. Normally, noises are a great barrier for the performance of conventional methods. In this scope, LDR-HE presents a scalable and successful contrast enhancement operation in the frequency domain. The modified HWT in the presented system removes this barrier providing that the noise points are minimized in low dynamic range. This type of a minimization cannot typically be performed by filtering in the spatial domain. With the help of LDR-HE, stretching noises are prevented.

Sharp ripples of signals in the frequency domain cannot be naturally detected in histogram and in spatial domain. This facility of detecting sudden changes in the frequencies only exists in the frequency domain. Therefore, in the presented method designed in the low dynamic range, high ripples of the frequencies are decreased whereas low frequencies are increased. Also, the mean value of intensities remains constant; however, the standard deviation gets lower. It has been analyzed, both mathematically and experimentally, that this proposed method produces an adjustable and controlled contrast enhancement with less error rate. LDR-HE might be recognized as a global, controlled, and adaptive HE method which enhances the contrasts with a lower time complexity. High frequency peaks are manipulated by an alpha parameter in order to eliminate the existent noises. The color intensities in all parts of the input image are manipulated by the proposed strategy. After the inverse transformation, LDR-HE also tries to preserve brightness as much as possible.

The primary purpose of this study is to prove how HWT is applicable on histograms in the frequency zone which can produce dynamic range reduction and contrast enhancement. Similarly, the proposed model enhances the picture quality by keeping the originality similar. In this system, HWT strengths the signal. Moreover, this paper proposes a novel modification technique while reducing the computational complexity. The main contribution of this study is that LDR-HE enhances image contrasts without any intensity saturation, noise amplification, over enhancement, and error rates. Extensive experimental studies indicate that this proposed scheme performs well in detail preservation and noise suppression.

In the proposed system, the DCT (discrete cosine transform) has been previously tested instead of HWT. Similar experimental results have been achieved with DCT. The point in this study is not related to the wavelet transform model. The structure of the alpha parameter in the coefficient of variations which is suited to the scheme is the key point in the presented HE operation.

5 Conclusion

In this study, an image enhancement approach by applying the Quantized Discrete Haar Wavelet Transform (Q-DHWT) on histogram equalization is presented. The main contribution of the proposed method is to enhance the image quality while equalizing the histogram in the low dynamic range (LDR) by suppressing the possible noises located in the high dynamic range (HDR). As a new contribution, a fast and scalable improvement framework is constructed by using a wavelet transform in order to eliminate naturally existing noises. Furthermore, the proposed model LDR-HE improves the diversity of color intensities throughout the image simultaneously and adequately. Both the qualitative and quantitative experimental studies indicate that histogram equalization in LDR-HE generates satisfactory results. Additionally, the suit of the image assessment metrics is employed in benchmarking studies in order to demonstrate the performance comparisons. The empirical studies display the proposed method overwhelms the common methods in the literature by providing more uniform distributions in the histogram with a scalable system.

Acknowledgements The introduced LDR-HE model has been implemented in the MATLAB R2020a and Java Processing v3.3.7 platforms. All of the implemented original codes, the benchmark datasets, the output images in both gray scale and RGB formats, the entire experimental results in terms of extra metrics, the full comparative lists in spreadsheet tables might be publicly downloaded from the web address [34] for the purpose of examinations, assessments, further studies, and citations. There is also a straightforward illustration by presenting an 8-bit scenario to make the procedures of the proposed method clearer. I would also like to express my deepest appreciations to the editor and anonymous reviewers whose creative comments helped improve and clarify this article.

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