Turk J Public Health
Available online at: http://dergipark.org.tr/tjph/
doi:10.20518/tjph.1028261

TURKISH JOURNAL OF PUBLIC HEALTH

ORIGINAL ARTICLE / ORİJİNAL MAKALE

A statistical analysis of COVID-19 pandemic based on the temporal evolution of entropy in different countries

COVID-19 pandemisinin farklı ülkelerdeki zamana bağlı entropi değişimine dayalı istatistiksel analizi

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Received: 25.11.2021, Accepted: 13.03.2022

ABSTRACT

Objective: Currently the Covid-19 pandemic is studied with great expectations by several epidemiological models with the aim of predicting the future behaviour of the pandemic. Determining the level of disorder in the pandemic can give us insight into the societal reactions to the pandemic the socio-economic structures and health systems in different countries. **Methods:** We perform a statistical analysis of Covid-19 pandemic using an entropy measure. For this, the Boltzmann-Gibbs-Shannon (BGS) entropy method is applied to the daily case data and the predictability in the covid-19 pandemic is discussed based on its entropic behaviour. The BGS entropy of the time evolution of daily cases in weekly groups from the beginning of the pandemic to 29 August 2021 in the UK, Germany, France, Italy, and Spain, Turkey, Russia and Iran are calculated and the given countries are classified by the predictability of the spread of the pandemic. **Results:** There is a clear difference in the predictability of the pandemic between the European countries and Turkey, Russia, and Iran. It is also observed that the vaccination programs and the Covid-19 variants of concerns; 20I/501Y.V1, 20H/501.V2, 21A/S:478K and 20J/501Y.V3 have effected the predictability of the pandemic in given countries are observed. **Conclusion:** The BGS entropy-based approach to determine the disorder in the time evolution of daily cases of the Covid-19 pandemic is effective and the results can be beneficial for comparison of the country classifications generated by the epidemiological models of this pandemic system.

Keywords: COVID-19, Epidemiology, Statistics, Entropy.

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Cite This Article: Yılmaz N., Akıllı M., Akdeniz KG. A statistical analysis of COVID-19 pandemic based on the temporal evolution of entropy in different countries. Turk J Public Health 2022;20(2):235-243

ÖZ

Amaç: Covid-19 pandemisinin Dünya üzerinde ilerlemesindeki davranışlarını anlama konusunda büyük bir beklenti ile farklı epidemiyolojik modeller kullanılarak çalışmalar yapılmaktadır. Pandeminin yayılmasındaki düzensizlik seviyeleri arasında bir karşılaştırma yapmak, ülkelerin pandemiye karşı toplumsal reaksiyonu, sosyo-ekonomik yapıları ve sağlık sistemleri hakkında da bilgi verebilir. **Yöntem:** Bu çalışmada, entropik ölçüt kullanarak Covid-19 pandemisinin istatistiksel analizini yapılmaktadır. Bunun için, günlük vaka sayılarınan oluşturulan verilere Boltzmann-Gibbs-Shannon (BGS) entropi metodu uygulanarak, Covid-19 pandemisinin entropik davranışına göre düzensizliği ve öngörülebilirliği belirlenmektedir. Birleşik Krallık, Almanya, Fransa, İtalya ve İspanya, Türkiye, Rusya ve İran'da pandemi başlangıcından 29 Ağustos 2021 tarihine kadar gerçekleşen günlük vaka sayıları, haftalık gruplara ayrılarak BGS entropi değerleri hesaplanmakta ve bu ülkeler pandemi yayılımının öngörülebilirliği konusunda sınıflandırılmaktadır. **Bulgular:** Pandeminin öngörülebilirliğinin beş büyük Avrupa ülkesi ile Turkiye, Rusya ve İran arasında farklılık gösterdiği tespit edilmektedir. Ayrıca, ülkelerin aşı programlarının ve 20I/501Y.V1, 20H/501.V2, 21A/S:478K, 20J/501Y.V3 varyantlarının incelenen ülkelerde pandeminin yayılmasındaki öngörülebilirliğe etki ettiği gözlenmektedir. **Sonuç:** BGS entropisi kullanılarak Covid-19 pandemisi günlük vaka sayılarının düzensizliğini belirlemenin, ülkeler arasında pandemi yayılımının karşılaştırmasında etkili olduğu görülmektedir ve elde edilen sonuçların pandemik sistem için epidemiyolojik modeller kullanılarak yapılan ülkelerin sınıflandırması çalışmalarında kullanılabileceği görülmektedir.

Anahtar Kelimeler: COVID-19, Epidemiyoloji, İstatistik, Entropi.

Introduction

Covid-19 Pandemic

Covid-19 is an infectious disease caused by a coronavirus, which was discovered at the end of 2019. It is a respiratory illness that is more likely to affect elder people and people with underlying medical conditions .[1]

There are many unknown factors in the prevention of its spread and finding better ways of treatment. Also, the measures to prevent the spread of the disease have drawbacks on the economic activity and social life. [2]

It has been a year and a half since Covid-19 was declared a pandemic by the World Health Organisation (WHO) on 11 March 2020. Currently, the spread of the virus has entered the fourth wave in Europe and in surrounding geography. All the efforts have recently focused on preventing more waves of the pandemic and minimize the effects of the pandemic in all aspects of life while lifting most restrictions and abapting social and economic life in a new normal. [3]

The vaccination is a major milestone in the fight against the pandemic. The UK was the first country to start a vaccination program on 8 December 2020. Since then, there have been emergency vaccination programs in use in many countries around the world with different approaches to vaccination and varying vaccination rates. One of the vaccines, the BNT162b2 mRNAvaccine has also been fully approved by the U.S. Food and Drug Administration and European Medicines Agency. [4-8]

On the other hand, a number of different variants have emerged around the world due to mutations of the virus. Among all the variants, four of them are currently known to be more contagious mutants of the virus and are classified as variants of concern by WHO. These variants are the Alpha variant that was first detected in the UK (20I/501Y.V1), the Beta variant that was first detected in South Africa (20H/501.V2), the Delta variand that was first detected in India (21A/S:478K), and the Gamma variant tah was first detected in Japan/Brazil (20J/501Y.V3). [9]

Therefore, it is also crucial to analyze the effects of the virus variants on the predictability of the pandemic.

In this work, we consider a statistical method to analyze the disorder and predictability of the Covid-19 pandemic. For this, we calculate the Boltzmann-Gibbs-Shannon (BGS) entropy and Maximum entropy of Covid-19 daily case data in weekly groups from 8 countries: The big five European countries (UK, Germany, France, Italy, and Spain) and three Eurasian countries (Turkey, Russia, and Iran). They are among the countries most affected from the pandemic along with USA, India and Brazil. We select the given 8 countries in our statistical analysis due to their close geographical proximity and having relatively close population size. First, we discuss the relation between covid-19 pandemic system and disorder, as the main purpose of this work is to compare predictability of the pandemic in different countries by the disordered characteristics. Then, we outline the BGS entropy that we apply to the time evolution of daily case data to distinguish the degree of disorder of the pandemic among the analyzed countries. In the results section, we plot the entropy of the daily case data in weekly groups for the pandemic in each country from the beginning of the pandemic to 29 August 2021. Lastly, we discuss the differences in the disorder of the pandemic in different countries. We especially underline the differences between the disorder in the European countries and Turkey, Russia, and Iran. We also highlight the possible effects of the variants of concerns on the disorder of the pandemic. Furthermore, we probe the effects of the vaccination programs in the given countries on the disorder of the pandemic. We then provide concluding remarks on the comparision of country classifications of the pandemic.

Disordered Systems and Covid-19 Pandemic

The disordered and complex systems evolve dynamically with unpredictable long-term behaviour. When a system changes its state, this change is non-linear as the response of the system's variables to its environment is not directly proportional to time. Therefore the disordered and complex systems are categorically non-linear. Thus, small environmental changes may have major effects on the behaviour of the system and the future state of the system depends on its past history as well as on its present state.[10] The study of disordered and complex systems provides a comprehensive, analytical, and cross-disciplinary perspective in the analysis of a wide variety of systems in many scientific fields such as physics, biology, medicine, psychology, engineering, social sciences, and finance.[11]

The health of a population can be considered as a disordered and complex system that the interactions between the variables that determine the spread of diseases in populations are often non-linear and that the spreading of a pandemic evolves on a complex network.[12,13]

Since January 2020, various epidemiological models based on complexity science and statistics have been put forward in understanding the disordered dynamics in the covid-19 pandemic and predicting its spread.[14-22]

Entropy is one of the most popular methods to determine the disorder and complexity of a system and different entropy measures are widely applied in diverse fields to quantify the disorder of dynamical systems.[23-27]

In this work, we perform a statistical analysis the Covid-19 pandemic using an entropy measue. We calculate the BGS entropy of the Covid-19 daily case data in weekly groups and discuss the disorder and the predictability in the covid-19 pandemic based on its entropic behaviour.

Methods

Boltzmann-Gibbs-Shannon Entropy

Boltzmann and Gibbs showed that the entropy of a system is linked to the microstates of the system in statistical thermodynamics.[28,29] The concept of entropy has gained general recognition with the introduction of statistical mechanics and later with the emergence of the information theory.[30-34]

Entropy in information theory, which was introduced by Shannon is directly analogous to entropy in statistical thermodynamics. The Boltzmann-Gibbs- Shannon (BGS) entropy form is defined by

$$S_{BGS}=-k\sum^{w}_{(i=1)}p_i \ln p_i \qquad (1)$$

Here, W is the total number of (microscopic) states, represents the probability of the ith state of the system. k is a positive constant related to the unit of entropy measurement. In thermodynamics, k corresponds to the Boltzmann constant $k_B = 1.38\ 10^{-23}$ J/K . In information theory $k = 1$.

Maximum Entropy

The entropy of a system becomes maximum when all the probabilities in the system are equal, hence the level of disorder is maximum. So, the aim of the maximum entropy principle is to choose the distribution with maximum entropy among all the possible distributions. S_{BGS} is maximum when the microstates are equiprobable, $p_i = 1/W$, as a consequence,

$$S_{BGS\ max} = k\ln(W) \qquad (2)$$

For the BGS entropy calculation of the daily case data of weekly groups as performed in the paper; the positive constant related to the unit of entropy measurement, $k = 1$, the number of states of the system, $W = 7$, and the resulting maximum entropy,

$S_{BGS\ max} = 1.95$

Results

Covid-19 data from 8 countries were used in the BGS entropy and the maximum entropy calculation.[35]

The data from the big five European countries (UK, Germany, France, Italy, and Spain) were used for the predictability classification in Europe. And the data from Turkey, Russia, and Iran were also used to study the difference in the predictability of the pandemic in those countries and Europe.

We analysed the daily case data in weekly groups to determine the probability distribution of the Covid-19 cases for each week. The reason we used the weekly probability distribution is that in modern society most of the economic and social interactions involve weekly cycles.

In this work, we performed the computation of BGS entropy and maximum entropy using Matlab program. Below are the daily case data, weekly BGS entropy and maximum entropy graphs from the beginning of the pandemic to 29 August 2021 (Figures 1-8).

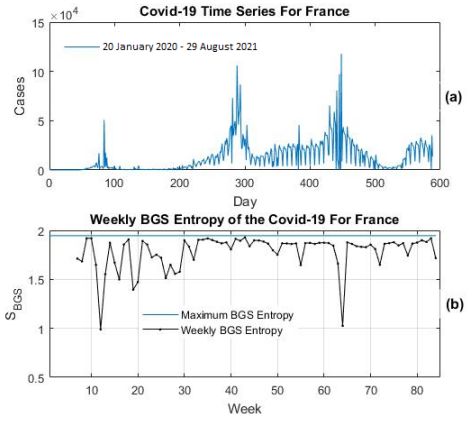

Figure 1 (a) is the daily cases in France from 20 January 2020 to 29 August 2021.
(b) represents the BGS Entropy and Maximum Entropy of the daily cases data in a weekly probability distribution.

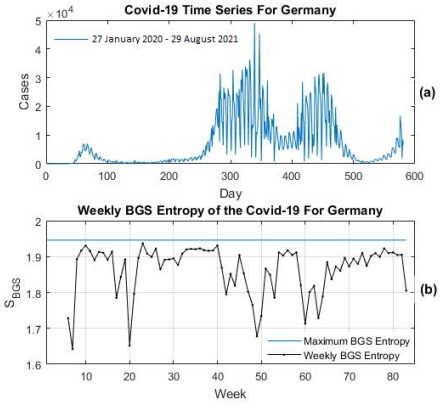

Figure 2 (a) is the daily cases in Germany from 27 January 2020 to 29 August 2021. (b) represents the BGS Entropy and Maximum Entropy of the daily cases data in a weekly probability distribution.

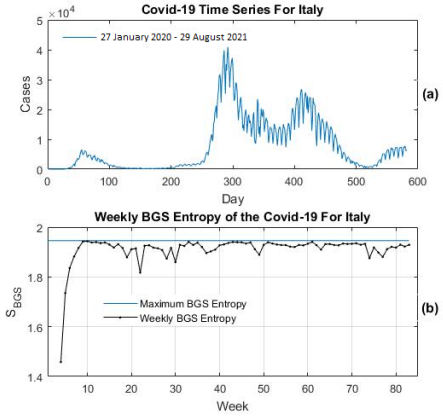

Figure 3 (a) is the daily cases in Italy from 27 January 2020 to 29 August 2021. (b) represents the BGS Entropy and Maximum Entropy of the daily cases data in a weekly probability distribution.

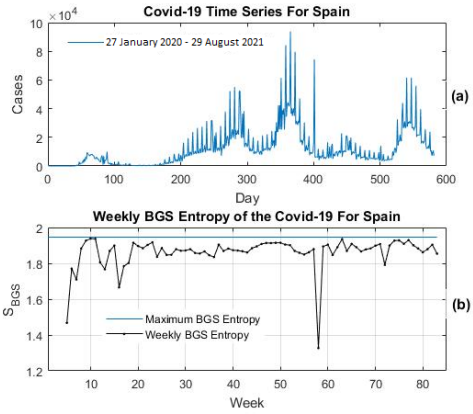

Figure 4 (a) is the daily cases in Spain from 27 January 2020 to 29 August 2021. (b) represents the BGS Entropy and Maximum Entropy of the daily cases data in a weekly probability distribution.

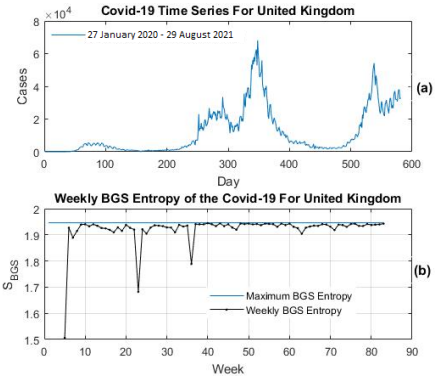

Figure 5 (a) is the daily cases in the UK from 27 January 2020 to 29 August 2021. (b) represents the BGS Entropy and Maximum Entropy of the daily cases data in a weekly probability distribution.

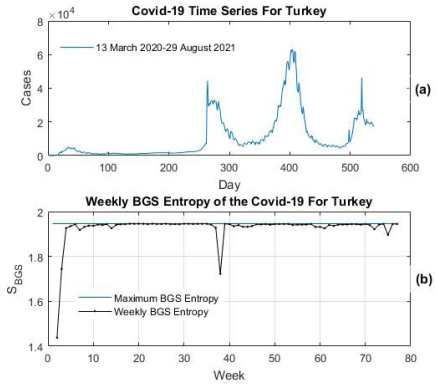

Figure 6 (a) is the daily cases in Turkey from 13 March 2020 to 29 August 2021. (b) represents the BGS Entropy and Maximum Entropy of the daily cases data in a weekly probability distribution.

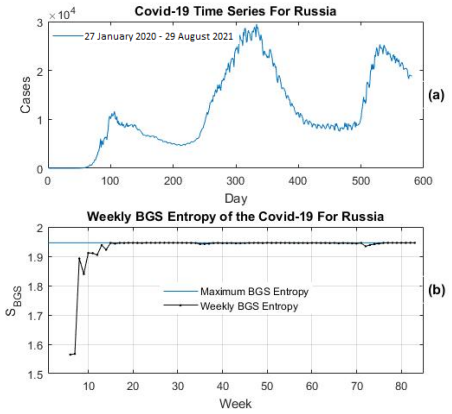

Figure 7 (a) is the daily cases in Russia from 27 January 2020 to 29 August 2021. (b) represents the BGS Entropy and Maximum Entropy of the daily cases data in a weekly probability distribution.

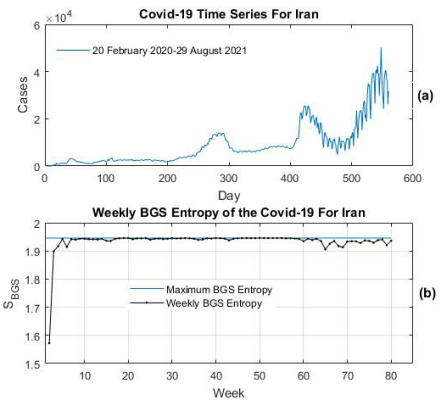

Figure 8 (a) is the daily cases in Iran from 20 February 2020 to 29 August 2021. (b) represents the BGS Entropy and Maximum Entropy of the daily cases data in a weekly probability distribution.

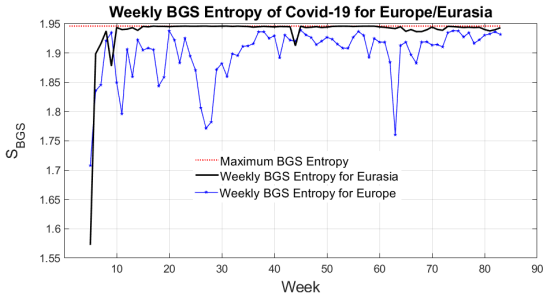

Figure 9 represents the BGS Entropy and Maximum Entropy of the mean daily cases data in a weekly probability distribution for the five Europen and three Eurasian countries from 27 January 2020 to 29 August 2021.

The comparision of the disorder and the predictability of the pandemic in the countries based on the BGS entropy graphs in each country from the beginning of the pandemic to 29 August 2021 (Table 1).

Table 1: Country classification on the predictability of the spread of the COVID-19.

	Level of Disorder Lower to Higher (Level of Predictability Higher to Lower)
Germany	1st
France	1st
Italy	3rd
Spain	4th
UK	5th
Turkey	6th
Russia	6th
Iran	6th

Discussion

Since the covid-19 was declared as a pandemic by WHO on 11 March 2020, countries around the world started to adopt various measures to prevent the spread of the virus mostly by limiting interaction of people. The second phase of the fight with the virus started on 8 December 2020 with vaccination programs. At the same time, the motations of the virus made containing the pandemic more difficult. Variants of the virus, the vaccination programs and limiting interaction of people are some of the variables in the pandemic system that effect the rate of the spread or the virus. The response of each variable in the system to the environment is not directly proportional to time so that the covid-19 pandemic system evolve dynamically with unpredictable long-term behaviour.[36]

Hence, as our results also indicate, the covid-19 pandemic can be considered a disordered and complex system.

In this work, we calculated the BGS entropy of the Covid-19 daily case data in weekly groups and investigated the disorder and the predictability in the covid-19 pandemic based on its entropic behaviour. The BGS entropy of Covid-19 pandemic case data increase to the level of the maximum entropy in all countries analyzed, during the peak of the first wave of the pandemic and stays very close to the maximum entropy throughout the first, second and third wave of the pandemic (Figures 1-9). This result underlines the strong disorder and complexity in the Covid-19 pandemic system.[13]

The BGS entropy graphs of the big five European countries, demonstrate that the disorder is lower to higher in the order of Germany, France, Italy, Spain, and the UK with marginal differences between them (Figures 1-5). Therefore, the predictability of the pandemic is the highest in Germany alongside France and lower in Italy, Spain, and the lowest in the UK as shown in table 1.

This could also indicate the transmission of the virus has been managed better in France and Germany than in Spain, Italy, and the UK.

The BGS entropy, hence the disorder in the UK daily case data is even higher than the previous weeks after the 48th week, almost equal to the maximum entropy (Figure 5). During the same period, the BGS entropy, hence the disorder in the other four European countries has a slight decrease (Figure 1-4). Considering that the UK is the most affected country by the Alpha variant of the virus (20I/501Y.V1) during the second wave of the pandemic, this may indicate the role of the Alpha variant (20I/501Y.V1) on the increased disorder of pandemic in the UK.[6]

Also, the level of disorder in the the third wave is similar to the first and second waves of the pandemic in all analyzed countries. This may indicate that, the new variables in the pandemic system such as the vaccination and the variants of concerns, especially the Delta variant (21A/S:478K) may have balancing effects on the pandemic system's disorder.[37]

Furthermore, it can be observed from the daily case data of the pandemic that the cases were lower in all countries during the third wave of the pandemic (Figure 1-8). However, the level of BGS entropy has not changed over the same period. This suggests that the overall decrease in the daily case data of the pandemic does not yet indicate any order in the stsyem as the predictability of the pandemic stays very low.

The interesting sharp down-peaks are observed in the BGS entropy of Germany (3), France (2), Spain (2), and the UK (2) and Turkey (1). No sharp down-peak is observed in Italy, Iran and Russia. Apart from this, the BGS entropy of all the European countries slightly fluctuates but the level of fluctuation is small.

Independent of the ever changing prevention mesures and the variant of concerns, the BGS entropy of Turkey, Russia, and Iran stay at a level with maximum entropy throughout the pandemic. A different picture emerges in the European countries where the BGS entropy is slightly lower than the maximum entropy (Figure 9). This shows the maximum disorder and minimum level of predictability in those three countries. And again, in the three Eurasian countries there is no change in the BGS Entropy during the pandemic apart from one sharp down-peak in Turkey in the 45th week compared to the fluctuating nature of the BGS entropy in the European countries. This may also indicate lower reliability in the provided daily case data in the three Eurasian countries. The difference in the level of disorder of the pandemic in the five European countries and the three Eurasian countries can be further investigated by social scientists and economists as well as medical scientists.

Conclusion

we performed a statistical analysis of Covid-19 pandemic using the BGS entropy measure. The BGS Entropy offers a different perspective for observing the progress of the covid-19 pandemic through its disorder and helps determine how predictable the future behaviour of the pandemic is in different countries. Moreover, measuring the level of disorder in the pandemic can also determine the reliability of an epidemiological model to predict the future behaviour of the pandemic. Thus, covid-19 BGS entropy calculation can be used in the comparison of the outcomes generated by various epidemiological models for the country classifications of the spread of the pandemic. The results can also provide information for understanding the relation between the socio-economic structures and the disorder and predictability of the covid-19 pandemic system.

Acknowledgement
The authors thank Assoc. Prof. Dr. Gülistan Çiğdem Yalçın for valuable discussions on complex characteristics of Covid-19 pandemic.

Ethıcal Declaration:
Ethics committee approval was not obtained for this study, as the study does not involve human participants and/or animal subjects.

Financial Support:
No funding was received for this study.

Conflict of Interest:
The authors declare no conflict of interest.

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